Impulse Technique for Structural Frequency Response Testing

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Impulse Technique for Structural Frequency Response Testing

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Structural frequency response testing, also known as "modal analysis," is becoming an integral part of the development and testing of a wide range of industrial and consumer products. It is an essential tool for the definition and solution of many types of structural dynamics problems, such as fatigue, vibration, and noise. This article discusses one of the most useful techniques for experimental structural frequency response testing — one based upon excitation of the structure with an impulsive force. In many situations, this is the simplest and fastest of the various techniques commonly used today. However, the nature of the excitation and response signals in the impulse technique requires special signal processing techniques if accurate frequency response measurements are to be obtained. This article discusses the application of the impulse technique and reviews the special problems encountered in practice and the techniques that have been developed for dealing with those problems.

Knowledge of the dynamic characteristics of structural elements often means the difference between success and failure in the solution of complex noise and vibration problems. The effects of structural resonances — conditions of relatively low dynamic stiffness — can lead to seriously reduced effectiveness of isolation elements and result in significantly increased dynamic response of sound radiating or vibration exposure elements. Quantitative knowledge of the frequencies, damping, and mode shapes associated with structural resonances aids in understanding how forces are generated and transmitted throughout mechanical systems and allows intelligent evaluation of various noise and vibration control modifications and treatments. The determination of the resonance characteristics of structures is termed "modal analysis." The purpose of this paper is to review in detail one particularly useful technique for experimental modal analysis, a technique employing the application of an impulsive force to the structure.

In two previous papers, the theory of modal analysis was reviewed and a number of techniques for experimental modal analysis were discussed, including swept-sine excitation, pure-random excitation, pseudo-random excitation, periodic-random excitation, and various forms of transient excitation.[1,2] The impulse technique falls into the class of transient excitation. It deserves particular attention because, for a wide range of structures, it is the simplest and fastest technique for obtaining good estimates of the required frequency response information. There are, however, a number of errors that can occur in the application of the impulse technique and there are certain types of structures for which the impulse technique is ill-suited. The major errors encountered in the application of the impulse technique will be discussed along with the signal processing and experimental techniques applicable to impulse testing.

Theory

Frequency Response Function. The measurement of the frequency response function is the heart of modal analysis. The frequency response function $H(f)$ is defined in terms of the single input/single output system, shown in Figure 1, as the ratio of the Fourier transforms of the system output or response $v(t)$ to the system input or excitation $u(t)$, Equation 1

$$H(f) = \frac{V(f)}{U(f)} \qquad 1$$

Where $V(f)$ = Fourier transform of system output $v(t)$
$U(f)$ = Fourier transform of system input $u(t)$.

The only requirements for a complete description of the frequency response function are that the input and output signals be Fourier transformable, a condition that is met by all physically realizable systems, and that the input signal be non-zero at all frequencies of interest. If the system is nonlinear or time-variant, the frequency response function will not be unique, but will be a function of the amplitude of the input signal in the case of a nonlinear system and a function of time in the case of a system with time-varying properties.

The frequency response function may be computed directly from the definition as the ratio of the Fourier transforms of the output and input signals. However, better results are obtained in practice by computing the frequency response function as the ratio of the cross-spectrum between the input and output to the power spectrum of the input, Equation 2. This relationship is derived by multiplying the numerator and denominator of the right-hand side of Equation 1 by the complex conjugate of the input Fourier transform.

$$H(f) = \frac{G_{uv}(f)}{G_u(f)} \qquad 2$$

where $G_{uv}(f) = U^*(f)\,V(f)$, cross-spectrum between $u(t)$ and $v(t)$
$G_u(f) = U^*(f)\,U(f)$, power spectrum of $u(t)$
U^* = complex conjugate of $U(f)$

The usefulness of this form of the frequency response function can be seen by considering the practical single input/single output measurement situation illustrated in Figure 2, where $m(t)$ and $n(t)$ represent noise at the input and output measurement points, respectively.

The measured frequency response function $H'(f)$ is given by the expression:

$$H'(f) = \frac{Y(f)}{X(f)} = \frac{V(f) + N(f)}{U(f) + M(f)} \qquad 3$$

where the upper case letters denote the Fourier transform of the corresponding time domain signals.

In this form, the measured frequency response will be a good approximation of the true frequency response only if the measurement noise at both the input and output measurement points is small relative to the input and output signals. Multiplying the numerator and denominator of the right-hand side of Equation 3 by the complex conjugate of $X(f)$ yields

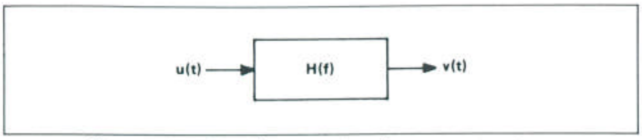

Figure 1 — Single input/single output system.

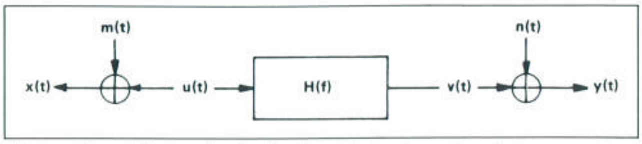

Figure 2 — General single input/single output measurement situation.

$$H'(f) = \frac{G_{uv}(f) + G_{un}(f) + G_{mv}(f) + G_{mn}(f)}{G_u(f) + G_{um}(f) + G_{mn}(f) + G_m(f)} \qquad 4$$

Now, if the measurement noise signals $m(t)$ and $n(t)$ are noncoherent with each other and with the input signal $u(t)$, then the *expected value* of the cross-spectrum terms involving m and n in Equation 4 will equal zero, yielding

$$H'(f) = \frac{G_{uv}(f)}{G_u(f) + G_m(f)} = \frac{H(f)}{1 + \left(\frac{G_m(f)}{G_u(f)}\right)} \qquad 5$$

where $H(f)$ is the desired true frequency response function.

Thus, if the noise-to-signal ratio at the input measurement point $[G_m(f)/G_u(f)]$ is much less than 1, the measured frequency response will closely approximate the desired true frequency response function.

It should be pointed out here that there is an inherent bias error associated with the computation of the cross-spectrum and the magnitude of this bias error is inversely proportional to the number of averages in the computation. Thus, the greater the measurement noise, the greater the number of averages required to approach the expected value of the cross-spectrum between the input and the output measurement signals. With measurement techniques employing many averages, the bias error can usually be reduced to an insignificant level so that it is only necessary to minimize the noise in the measurement of the input signal. However, if there is significant measurement noise and only a few averages are used, then the computed values of the cross-spectrum terms involving the noise signals in Equation 4 can be large relative to the true cross-spectrum, with resulting large errors in the measured frequency response function. In general, only a few averages are used in the impulse technique; otherwise, one of its major advantages — its speed — is lost. Therefore, it is important to minimize measurement noise in both the input and output signals when using the impulse technique. The cross-spectrum bias error and its effects are discussed in more detail in Reference 3.

Coherence Function. There is another important reason for computing the frequency response function in terms of the cross-spectrum: it allows the computation of the coherence function between the input and output signals. The coherence function is defined by the equation

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_x(f)\, G_y(f)} \qquad 6$$

According to the definitions of the power spectrum and the cross-spectrum, the coherence function will be identically equal to 1 if there is no measurement noise and the system is linear. The minimum value of the coherence function, which occurs when the two signals are totally uncorrelated, is 0. Thus, the coherence function is a measure of the contamination of the two signals in terms of noise and nonlinear effects, with very low contamination indicated for values close to 1.

Since the cross-spectrum is included in the definition of the coherence function, the cross-spectrum bias error must be reduced to an acceptable level if a good statistical estimate of the coherence function is to be achieved. As stated above, the number of averages used in the impulse technique is usually not great enough to significantly reduce the bias error. However, the coherence function is still useful for indicating the importance of noise in the impulse technique. This is because noise in the signals causes variance in the value of the coherence function with frequency. This effect is illustrated in the section on measurement procedures.

Display of Frequency Response. The frequency response function is complex — that is, it has associated with it both magnitude and phase. Therefore, it can be displayed in a number of forms, including magnitude and phase versus frequency, real and imaginary magnitudes versus frequency, and imaginary magnitude versus real magnitude. Each of these types of displays has its own particular usefulness. The most common type of display for structural frequency response data is magnitude and phase versus frequency, with the magnitude and frequency plotted logarithmically. This type of display, with the magnitude in terms of compliance (ratio of displacement to force), is called a Bode plot. In this form of the frequency response function, resonances occur as peaks in compliance plots (points of maximum dynamic weakness) and all resonance peaks of equal damping have the same width regardless of resonance frequency. Lines of constant dynamic stiffness have zero slope, and mass-dominated frequency response lines have a −12 dB-per-octave slope. Figure 3 shows an example of a Bode plot of a measured frequency response function.

Resonances occur as nearly circular arcs in the complex plane (real versus imaginary plot) with frequency increasing in a clockwise direction around the arc. In the case of real normal modes (which occur in systems with relatively low damping and with resonances well-separated in frequency), each resonance arc is approximately tangent with, and lies below, the real axis and is symmetric about the imaginary axis when the frequency response is expressed as compliance. The complex plane plot is useful when certain types of analytical curve fitting operations are being performed on the frequency response data. Figure 4 shows the complex plane plot of the frequency response function shown in Figure 3.

The plots of the real and imaginary magnitudes of frequency response versus frequency are most useful when dealing with real normal modes. In this case the resonances will occur as peaks in the imaginary magnitude plot and the real magnitude will pass through zero at the resonance frequency when the frequency response is expressed as compliance. Figure 5 shows the real and imaginary plots for the data in Figure 3.

The frequency response characteristics of a structual element are determined by measuring a set of cross-frequency response functions as discussed in Reference 1. The cross-frequency response functions may be obtained by exciting at one location on the structure and measuring response at various locations, or by measuring the response

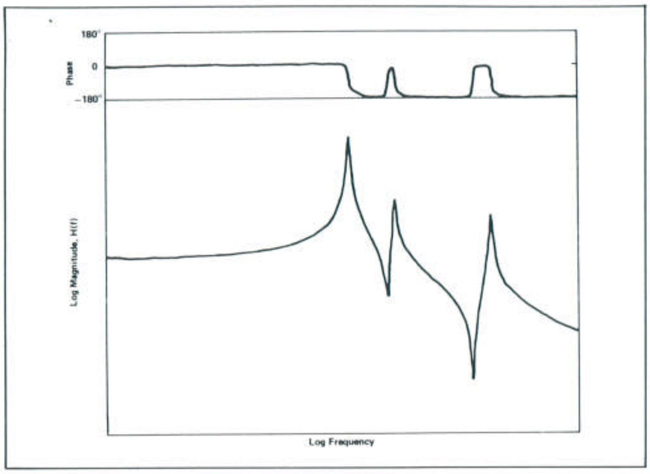

Figure 3 — Bode plot of typical frequency response function.

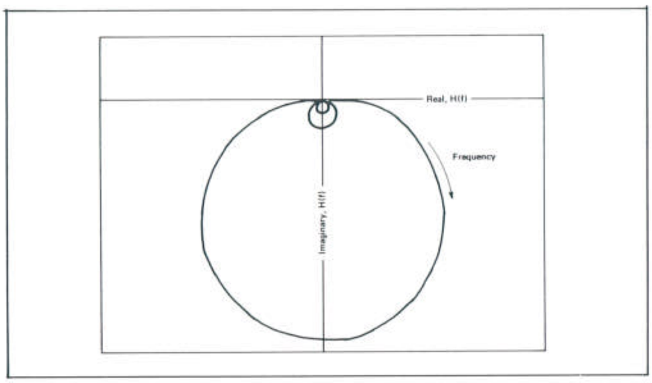

Figure 4 — Nyquist plot of typical frequency response function.

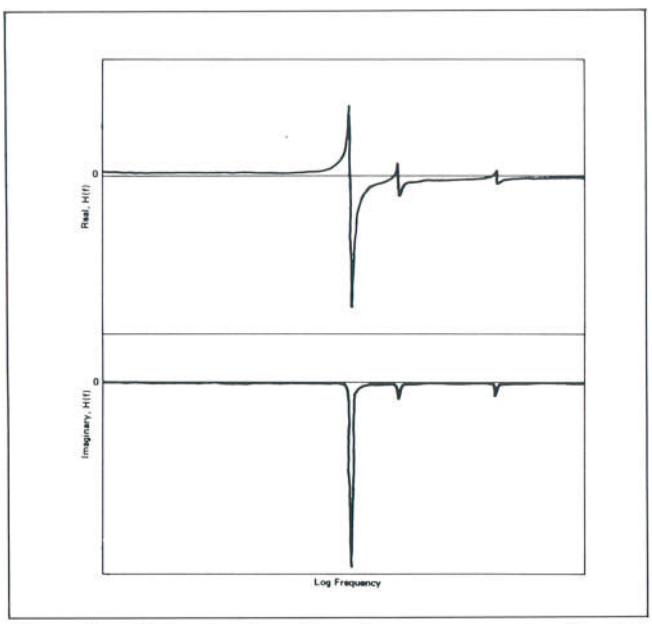

Figure 5 — Plots of real and imaginary components of typical frequency response function.

at a single location to excitation at various locations. The resulting frequency response functions comprise one column of the transfer matrix in the first case, and one row of the transfer matrix in the second case. Either set will, in general, completely define the modal characteristics of the structural element. In mathematical terms the set of frequency response functions yields the eigenvalues and eigenvectors, which are, in general, complex terms. The real part of an eigenvalue is the damping and the imaginary part is the frequency associated with a given resonance. Each eigenvector defines a resonance mode shape.

With real normal modes, each point on a structure is either exactly in-phase or exactly 180 degrees out-of-phase with any other point at the resonance frequency. Certain types of damping which are often encountered in practice will cause the eigenvectors to have non-zero imaginary components, resulting in complex mode shapes. When a mode is complex, the relative phase associated with a point on a structure is some value other than 0 or 180 degrees, with the result that node lines (lines of zero deflection) are not stationary. Precise description of complex modes requires that some type of analytical curve fitting technique be applied to the frequency response data.

Measurement of Frequency Response. The frequency response function of an operating system can be computed if the system input and output signals meet previously stated requirements of Fourier transformability and non-zero value, assuming the system input and response can be measured. However, in practice there are usually multiple inputs to the system — either several inputs at different locations or inputs in more than one direction at a given location. In the case of multiple coherent inputs, the complexity of the analysis is greatly increased. For this reason, and the difficulty of accurately monitoring operating inputs, frequency response measurements are usually made by applying the system input "artificially" through some type of exciter. It is in the form of the input signal and the way it is applied to the structure that the wide variety of frequency response testing techniques arises.

The usefulness of the impulse technique lies in the fact that the energy in an impulse is distributed continuously in the frequency domain rather than occurring at discrete spectral lines as in the case of periodic signals. Thus, an impulse force will excite all resonances within its useful frequency range. The extent of the useful frequency range of an impulse is a function of the shape of the impulse and its time duration. Figure 6 shows the frequency spectra of two square pulses of equal energy but different duration. For a square pulse the frequencies of the zero crossings are at integral multiples of the inverse of the time duration of the impulse, illustrating the very important inverse relationship between the time duration of an impulse and its frequency content.

The useful frequency range of an impulse is also a function of the shape of the impulse. Figure 7 shows three

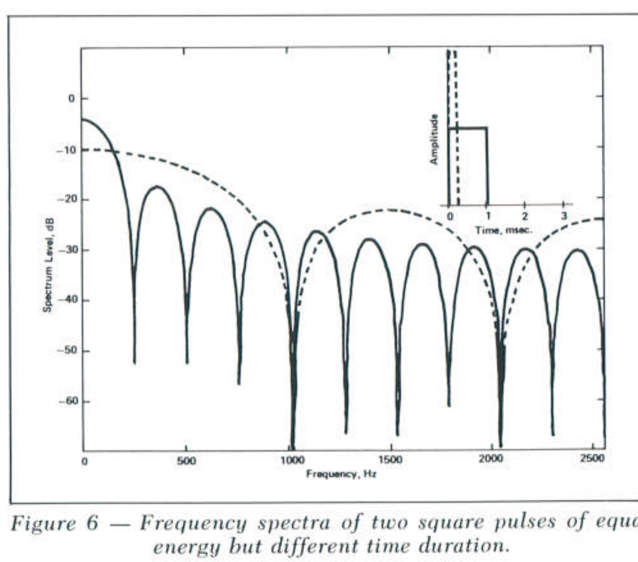

Figure 6 — Frequency spectra of two square pulses of equal energy but different time duration.

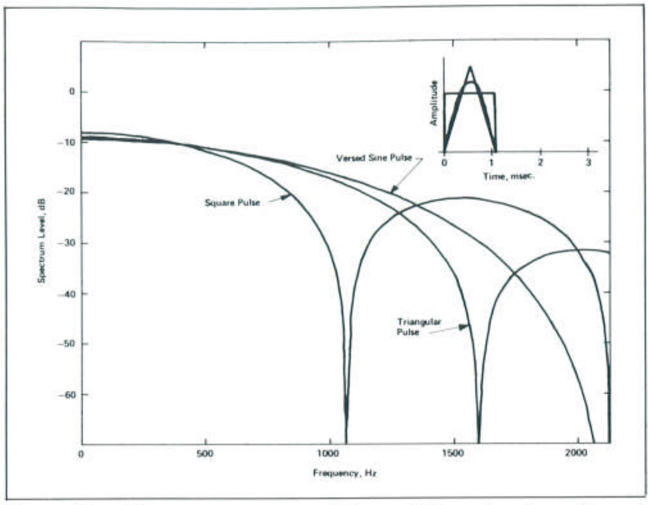

Figure 7 — Frequency spectra of three different pulses of equal energy.

different pulses of equal energy and time duration and their corresponding frequency spectra. By varying the weight and hardness of an impacting device and the manner in which the impact is applied, the shape and time duration of the impulse produced can be varied to suit the measurement requirements. Such practical aspects will be further discussed in the section on experimental measurement techniques.

Nonlinearities in Structures

Excitation of a nonlinear system by a pure-random signal will yield the best estimate (in a mean-square sense) of the linear system response. Excitation by a pure sine wave is also useful for studying nonlinear systems because it allows precise control of the input spectrum level. However, the impulse technique, because of its very high ratio of peak level to total energy, is particularly ill-suited for testing nonlinear systems. Therefore, it is important to understand the various types of nonlinearities that can occur in structural systems and to be able to recognize nonlinearities in measured frequency response functions.

One of the most common types of nonlinearities encountered in structures is that due to clearance between parts. This type of nonlinearity is frequently encountered, for example, when testing gear systems and shafts mounted in bearings. The effects of this type of nonlinearity on measured frequency response functions when using impulse excitation are poor estimates of static stiffness values and poor repeatability of the frequency response estimates. Also, the apparent damping in the estimates will be greater than the actual examples.

The best method of dealing with this type of nonlinearity is to preload the system to take up clearances. Care must be taken when this is done, however, because any preload will change the boundary conditions of the structure and can itself lead to erroneous frequency response estimates. The usual approach is to apply the preload through a very soft spring so that the resonances associated with the preload lie below the frequency range of interest.

Another type of nonlinearity that is frequently encountered is nonlinear damping. Nonlinear damping effects are usually associated with joints in the structure, where the damping is a function of the relative displacement at the joint. In general, the frequency response estimates obtained by the impulse technique will agree most closely with those obtained with a low level of continuous excitation. However, if the point of excitation is close to a location where nonlinear damping occurs, there will be high relative motion at that location, and the apparent damping in the measured frequency response will be high. In systems with low damping, this will give the measured frequency response a discontinuous appearance, due to the varying level of damping as the response to the impulse attenuates with time. This type of nonlinearity is illustrated in Figure 8, which shows frequency response measurements on a machine tool with different force excitation levels. The frequency response measurements were made with swept-sine excitation.

The third type of nonlinearity that commonly occurs in structures is load-sensitive stiffness, where the spring rate of elastic elements either increases or decreases with load. The most direct way to identify this type of nonlinearity is to measure frequency response as a function of static preload and observe the change in resonance frequencies. This type of nonlinearity is illustrated in Figure 9, which shows frequency response measurements on a pump with three different levels of preload.

Signal Processing

The particular characteristics of an impulsive force signal and the resulting structural response signal make the impulse technique especially susceptible to two problems: noise and truncation errors. While these problems occur to some extent with other frequency response testing techniques, their unique importance in the impulse technique requires special signal processing methods.

Force Signal. It was pointed out in the previous section that the usable frequency range for an impulse depends on the shape and time duration of the impulse. In order to insure that there is sufficient force over the frequency range of interest, it is necessary that the first zero crossing of the Fourier transform of the impulse be well above the maximum frequency of interest. For a given time duration the first zero crossing occurs at the lowest frequency for a square pulse. For that type of pulse the first zero crossing occurs at a frequency equal to the inverse of the time duration. A good rule of thumb, then, is to insure that the duration of the impulse is less than $2\Delta t$, where Δt is the sampling interval in the analog-to-digital conversion process. This would put the first zero crossing of the Fourier transform of a square pulse at the Nyquist folding frequency, and the first zero crossing of other pulse shapes above the Nyquist folding frequency.

The sample length is equal to $N\Delta t$ where N is the number of digital values in each sample. A typical value of N is 1024. Thus, the duration of the impulse is very short relative to the sample length. This means that the total energy of noise represented in the time-sample can be on the order of the energy of the impulse, even for high signal-to-noise ratios. The noise problem is further aggravated when employing the zoom transform, which yields increased resolution in a given frequency band by effectively increasing the sample length.

With other techniques, the effects of noise are reduced by averaging the power spectrum and cross-spectrum functions prior to the computation of the frequency response function. However, only a few averages are usually used in the impulse technique. Otherwise, the time advantage of the technique is lost. Therefore, special time-sample windows have been developed for the impulse technique.

At first thought it might seem appropriate to just set all time-sample values beyond the impulse to zero, since it is